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Sliding mode control of discrete time dynamical systems using the reaching law approach

PhD dissertation
(synopsis)

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Sliding mode controllers have first been proposed for continuous time systems in the middle of the 20th century. They are well known to be computationally efficient and insensitive with respect to disturbance that satisfies the matching conditions. However, since most modern control processes are applied digitally, a natural development was the introduction of discrete time sliding mode controllers. For discrete time plants, sliding mode control strategies ensure good robustness with respect to disturbance by driving the system representative point to a specified vicinity of the sliding hyperplane. Depending on the evolution of the state trajectory in the sliding phase, discrete time sliding mode controllers can be divided into two categories. Strategies from the first category, referred to as switching type, drive the system representative point to cross the sliding hyperplane in each step. Methods from the second category, called non-switching type, drive the representative point to a vicinity of the hyperplane without the necessity to cross it.

In this dissertation, seven new discrete time sliding mode control strategies based on the reaching law approach are proposed. In this approach, the desired evolution of the sliding variable is stated *a priori* in the form of a reaching law and then applied to design the control signal. This allows one to manage the evolution of the system state at all stages of the control process and bypass the often complex proof of stability. It has been demonstrated that all new reaching laws proposed in this dissertation improve robustness of the controlled system compared to commonly used sliding mode control strategies.

In the second chapter of the dissertation, four new discrete time reaching laws for conventional sliding variables s with relative degree one are introduced. The first two strategies aim to ensure switching type quasi-sliding motion, in which the system representative point is driven to cross the sliding hyperplane in each time instant. The first reaching law consists of two terms: a saturating function of the sliding variable and a constant switching term. It is expressed as

$$s(k+1) = g[s(k)]s(k) - \varepsilon \mathrm{sgn}[s(k)] + D(k) - D^{\mathrm{avg}} - D^{\delta}\mathrm{sgn}[s(k)], \tag{1}$$

where $g(s) = \min\{1, |s|/q\}$, $D(k)$ is the unpredictable effect of disturbance on the sliding variable (with mean D^{avg} and maximum deviation from the mean D^{δ}) and ε, q are positive constants. The objective of this strategy is to limit the convergence rate of the system representative point to the vicinity of the sliding hyperplane and confine it to that vicinity for all future time instants. Indeed, it is demonstrated that the representative point is confined to a specified quasi-sliding mode band and ensures switching type motion of the system representative point. The second reaching law aiming to ensure switching type quasi-sliding motion uses an exponential function of the sliding variable and has the following form

$$s(k+1) = h[s(k)]\{s(k) - \varepsilon \text{sgn}[s(k)]\} + D(k) - D^{\text{avg}} - D^{\delta}\text{sgn}[s(k)], \tag{2}$$

where $h(s) = 1 - \exp(-s^2/s_0^2)$ and s_0 is a positive design parameter. The goal of this strategy is to reduce maximum overshoot in switching type quasi-sliding motion for values of s close to zero. As a result, the system representative point is driven to a narrower quasi-sliding mode band than in other comparable strategies. It has been demonstrated that for both reaching laws (1) and (2) the absolute value of all state variables is bounded in the sliding phase. Moreover, it has been proven that when switching type quasi-sliding motion is ensured, the bounds of the state variables can be further reduced. Results concerning switching type reaching laws (1) and (2) have been published in international journals [8] and [3], respectively.

Further in the second chapter of this dissertation, two reaching laws ensuring non-switching type quasi-sliding motion have been proposed. Just like strategies (1) and (2) described above, the new reaching laws use conventional relative degree one sliding variables. The first reaching law uses the saturating function $g(s) = \min\{1, |s|/q\}$ and has the following form

$$s(k+1) = s(k) - \varepsilon g[s(k)]\text{sgn}[s(k)] + D(k) - D^{\text{avg}}. \tag{3}$$

The second reaching law incorporates the exponential function $h(s) = 1 - \exp(-s^2/s_0^2)$ and can be expressed as

$$s(k+1) = s(k) - s_0 h[s(k)]\text{sgn}[s(k)] + D(k) - D^{\text{avg}}. \tag{4}$$

Both reaching laws aim to reduce the magnitude of the switching term as the system representative point approaches the sliding hyperplane. This eliminates the need for changing the sign of the sliding variable in each step and allows one to drive the system representative point to a narrower vicinity of the sliding hyperplane than in the case of switching type strategies. Just like for reaching laws (1) and (2), it has been proven that non-switching strategies (3) and (4) limit the absolute value of all state variables in the sliding phase. The proposed non-switching reaching laws have then been used to control a particular class of discrete time plants representing inventory management systems with multiple suppliers. For such plants it has been demonstrated that:

- The control signal is always non-negative and upper bounded, which means that suppliers are never required to exceed their maximum transport capabilities or accept returned shipments.

- The amount of stored goods is upper bounded and, after a finite number of initial time instants, strictly positive. This implies that the available warehouse capacity is not exceeded and that the consumers' demand is always fully satisfied.

The third chapter of this dissertation is devoted to discrete sliding variables s_r with an arbitrary relative degree r. This is an important contribution, since the use of such variables in discrete time sliding mode control is a novel area of research. Sliding variables with arbitrary relative degree have an important property in that they are only affected by the control signal and matched perturbations from a number of time instants ago, equal to the relative degree. For such variables, three new reaching laws have been proposed with the aim of ensuring switching type quasi-sliding motion of the system. The first reaching law is designed for sliding variables with relative degree two. It is a generalization of classic strategy with the constant switching term and one proportional to the sliding variable. The new reaching law has the following form

$$s_2(k+2) = q^2 s_2(k) - q\varepsilon_2 \mathrm{sgn}[s_2(k)] - \varepsilon \mathrm{sgn}[s_2(k+1)] + D_2(k) - D_2^{\mathrm{avg}}, \tag{5}$$

where $D_2(k)$ is the effect of perturbations on the relative degree two variable, D_2^{avg} is its mean value, q, ε_2 are positive constants and $q < 1$. The proposed strategy ensures switching type quasi-sliding motion of the system while providing better robustness with respect to disturbance than its equivalent for relative degree one sliding variables. The second reaching law proposed in this chapter further generalizes strategy (5) in order to make it applicable to sliding variables with arbitrary relative degree r. It is expressed as

$$\begin{aligned} s_r(k+r) =& q^r s_r(k) - q^{r-1}\varepsilon_r \mathrm{sgn}[s_r(k)] - q^{r-2}\varepsilon_r \mathrm{sgn}[s_r(k+1)] - \ldots - \\ &- q\varepsilon_r \mathrm{sgn}[s_r(k+r-2)] - \varepsilon_r \mathrm{sgn}[s_r(k+r-1)] + D_r(k) - D_r^{\mathrm{avg}}, \end{aligned} \tag{6}$$

where q and ε_r are positive design parameters, $D_r(k)$ is the effect of disturbance on the relative degree r variable and D_r^{avg} is its mean. It has been demonstrated that in the sliding phase, the proposed reaching law ensures switching type quasi-sliding motion of the system and confines its representative point to a specified quasi-sliding mode band. However, ensuring that the sliding phase is reached in finite time can be complicated for sliding variables with an arbitrary relative degree. Therefore, in order to ensure that the sliding phase begins immediately at the start of the control process, a shifted, time-varying sliding hyperplane has been proposed. It has been demonstrated that as the relative degree of the sliding variable increases, the minimum quasi-sliding mode band width becomes smaller. This is an important property, since the absolute value of all state variables in the sliding phase is directly proportional to that width.

Further in the third chapter of the dissertation, a new reaching law for relative degree two sliding variables is proposed. This reaching law can be expressed as

$$\begin{aligned} s_2(k+2) =& g[s_2(k+1)]g[s_2(k)]s_2(k+1) - 0{,}5\{1+g[s_2(k)]\}\varepsilon \mathrm{sgn}[s_2(k+1)] \\ & - 0{,}5\{1-g[s_2(k)]\}\varepsilon \mathrm{sgn}[s_2(k)] + D_2(k) - D_2^{\mathrm{avg}} - D_2^{\delta}\mathrm{sgn}[s_2(k+1)], \end{aligned} \tag{7}$$

where $g(s) = \min\{1, |s|/q\}$ is the saturating function and $q > 0$. The objective of this function is to ensure a bounded convergence rate of the system representative point to the vicinity of the sliding hyperplane and ensure switching type quasi-sliding motion. Just like for the previous strategies proposed in this chapter it is demonstrated that the new method drives the system representative point to a narrower vicinity of the sliding hyperplane than its equivalent for relative degree one variables. As a result, the absolute value of all state variables in the sliding phase is smaller than for other commonly used methods.

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